16

Central Tendency and Variation

Measuring the Central Tendency of Data
Measuring the Variability of Data
Statistics and Parameters
The Standard (z-) Score

In the last chapter we considered a way to reduce a mass of numbers by creating a grouped frequency distribution and graphing it. The graph is a visual image of the data, and is an important first step in data analysis. In this chapter we develop basic concepts in *reducing data numerically.* A group of numbers has **two primary numerical characteristics.** The first is a *central point about which* they cluster, called the ***central tendency***. The second is *how tightly they cluster* about that point, called ***variability***.

Measuring Central Tendency

The central tendency of a group of scores is the numerical focus point of those scores. It refers to the point of greatest concentration of the set of numbers. There are three separate measures of central tendency. These are the *mode*, the *median*, and the *mean*.

The Mode

The mode is the *most frequently occurring score* in a set of scores.

82 82 83 83 84 85 86 87 87 87 88 90 95 99 99

The mode of the above set of numbers is 87 because it appears three times — more than any other number in the set.

82 83 84 86 87 88 88 89 90 91 91 92 94 97 98

There are two modes above. The numbers 88 and 91 both appear twice. This is a bi-modal (two modes) data set.

82 83 84 86 87 88 89 90 91 92 93 94 95 96 97

There is no mode for this distribution. No score occurs more frequently than any other. The ***mode*** is the ***most frequent score*** in a set of data.

The Median

The ***median*** is the ***middlemost score***. That is, it is the score that represents the exact halfway point through the data set. The median score divides the set of data into two equal halves. Half of the scores falls below the median, and half falls above it.

1 3 4 6 8 **9** 34 56 67 100 356

In the above set, it is the 6th score, or the number 9. Five scores fall below 9, and five scores fall above 9. We can calculate this score with the simple formula: **(N+1)/2** where N is the number of scores in the set. There are 11 scores (N=11) in the data set above. Using the formula, we compute (11+1)/2 = 6. The 6th score is the median, which is the number 9. The median of this data set is 9.

1 3 4 6 8 9 34 56 67 100 356
5 scores below median 5 scores above

Here's another example:

34 23 67 4 8 17 2 78 99 5 178 3 1678

First, we rank order the numbers from low to high.

2 3 4 5 8 17 **23** 34 67 78 99 178 1678

Applying the formula, we compute (13+1)/2 = 7. We are looking for the 7th score. The 7th score is the number **23**. 23 is the middle number, the median. Six scores fall above and six below this number.

Here's an example with an even number of scores:

1 2 3 4 5 6 7 8 9 10 11 12 13 14

In this case, there are two "middlemost" values. The median for this data set is the average of the two middlemost values. Add the two middle values together and divide by 2. In our case, (7+8)/2 = **7.5**. Notice that seven numbers fall below 7.5 (1-7) and seven numbers fall above it (8-14).

1 2 3 4 5 6 **7** **8** 9 10 11 12 13 14
_ *7.5*

The ***median*** is the ***middlemost score***.

The Arithmetic Mean

The ***mean*** is the ***average value*** of a data set and is the *best representation of the set of scores*. You have often computed test averages in your courses by adding together several test scores and dividing by the number of tests.

1 2 3 4 5 6 7 8 9 10

The mean is found by adding these ten numbers (N) together and dividing by ten (N). We can represent the procedure for computing a mean in a shorter form by using symbols. You were introduced to the **Σ** symbol (capital sigma) in chapter 14. The symbol **X** (capital "X" or "Y" or "L" or any English letter) refers to **scores**. The letter **N** refers to the number of scores. And finally, the Greek letter **μ** (pronounced *myoo*) represents the arithmetic mean of the scores. Using these letters to define the formula for the mean, we have the following:

$$\mu = \frac{\Sigma X}{N}$$

Read the above formula like this: ***"mu equals the sum of X divided by N."*** Or, in English, ***"the average value of a group of scores is the sum of those scores divided by the number of scores in the group."***

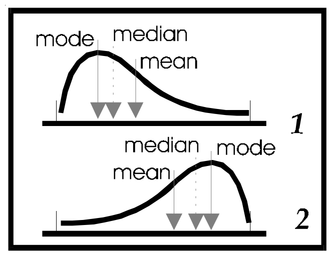
Let's use this formula on the following data set: ***10 23 17 5 64 28 3***

$$\mu = \frac{\Sigma X}{N} = \frac{10+23+17+5+64+28+3}{7} = \frac{150}{7} = 21.43$$

The **mean** score of 21.43 represents the ***average value*** of all the individual scores in the group, and is the most important measure of central tendency due to its use in statistical analysis.

Central Tendency and Skew

When a distribution is a normal ("bell-shaped") curve, all three measures of central tendency have the same value. If a distribution is skewed, the three measures differ in a predictable way. The mode will always equal the highest frequency value. The mean moves away from the mode *in the direction of the skew*. (Extreme scores pull the mean toward them).



In the positively skewed distribution at right ("1") the mean is greater than the mode.

In the negatively skewed distribution ("2") the mean is smaller than the mode. The median is always between mode and mean.

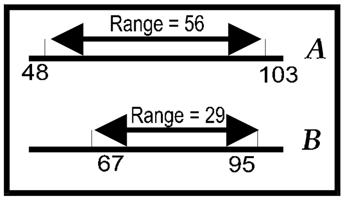
We have defined three measures of central tendency -- the mode, median, and mean -- and established the prominence of the mean. Now we turn to the second essential characteristic of scores -- variability.

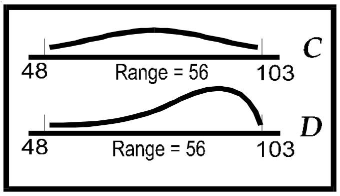
Measures of Variability

The second essential characteristic of a group of scores is variability. ***Variability*** is a measure of ***how tightly a group of scores clusters*** about the mean. Scores can be tightly clustered or loosely clustered about the mean. Scores that tightly cluster about the mean have lower variability. Scores that loosely cluster, that are more spread out from the mean, have higher variability. There are three measures of variability. These are ***range***, ***average deviation***, and ***standard deviation***.

Range

As we learned in the last chapter, the range of a group of scores is equal to the highest score minus the lowest score plus 1, or, **Range = $X_{max} - X_{min}$ + 1**. It is a crude





measure of variability, but is a useful first step in understanding a distribution. Let's look at an example.

Class A took a midterm examination in research. The highest score in the class was 103 and the lowest was 48. The range was 103 - 48 + 1 or 56 points. Class B is the same size and took the same exam. Their highest and lowest scores were 95 and 67 respectively. Their range was 95 - 67 + 1 or 29 points. Therefore, the scores of Class B have lower variability (more tightly clustered) than the scores of Class A.

The problem with range is that it tells us nothing of the dispersion of scores between the high and low points. Classes C and D have the same ranges, but have different dispersions of scores. One way of getting at the dispersion of scores throughout the whole distribution is to measure the deviation of each score from the mean — and then compute the average of all the deviations.

Average Deviation

A deviation score, symbolized by a lower case x, is the difference between a score (**X**) and the mean (**μ**) of the distribution. When you subtract the mean of a group of scores from a specific score, you compute the **deviation** of the score from the mean. Or, we can write this relationship more simply as x = X - μ.

The average deviation of a group of scores is computed by summing all the deviations in the group and dividing by N. Look at the following scores:

```
10     20     30     40     50
```

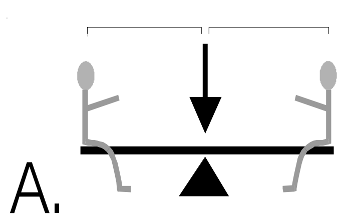
First, compute the mean of these scores: (150/5=**30**). Then compute the deviation scores (x) by subtracting the mean (30) from each score (X) like this:

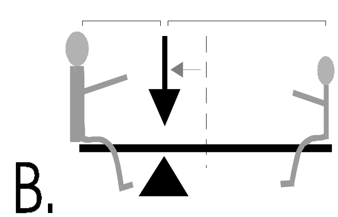
deviation scores (x)

raw scores (X)

score	-	mean	=	deviation
10	-	30		-20
20	-	30		-10
30	-	30		0
40	-	30		10
50	-	30		20
sum of deviations (Σx)			=	0

Notice that when we sum the deviations, we get 0 (Σx=0). Why is the sum of deviations equalled to zero? The mean is the balance point in a distribution. When two children of equal weight use a teeter-totter, the balance point is placed half-way between them, as in diagram A below left.





But when children of unequal weight use it, the board must be changed so that the balance point, or fulcrum, is closer to the heavier child. This is shown in B below right. Heavier weight plus shorter distance on one side of the board balances with the lighter weight and longer distance of the other. Another way of saying this is that for perfect balance, the moment of force (weight x distance) of one side equals the moment of force of the other. Subtract one from the other and the result is zero. This is what is meant in statistics when we say *the mean is the fulcrum of a group of scores*. Large deviations are like large distances from the fulcrum, and small deviations like small distances. (All scores "weigh" the same in this example). The sum of deviations on one side of the mean will always cancel out or balance the sum of deviations on the other side of the mean. Therefore, $\Sigma x=0$.

In order to compute average deviation, we must take the absolute values of the deviations. An absolute value, symbolized as $|x|$, equals the value of a number regardless of sign. So, the absolute value of -4 equals 4 ($|-4| = 4$).. By taking the absolute values of deviations, we make them all positive distances from the mean. Summing them, we produce a meaningful measure of "spreadedness" from the mean:

$$\text{Average Deviation} = \frac{\Sigma|x|}{N} = \frac{20+10+0+10+20}{5} = \frac{60}{5} = 12.0$$

The ***average deviation*** equals ***12***. But average deviation has some mathematical limitations that cause problems in more advanced procedures. A better measure of variability, which also reflects the dispersion of scores throughout a distribution, is the ***standard deviation***.

Standard deviation

The standard deviation has mathematical properties which make it, like the mean, much more useful in higher-order statistics. The procedure for standard deviation involves summing squared deviations (producing a value variously called ***the sum of squared deviations***, ***sum of squares***, and statistically, Σx^2, which is a fundamental component of many statistical procedures) in order to eliminate negative values. The pathway to standard deviation moves from *deviations* to the *sum of squares* to *variance* to *standard deviation*.

We'll look at two ways to compute sum of squares. The first, called the *deviation method*, clearly **illustrates what standard deviation means**. The second, called the *raw score method*, **is easier to use**. Both procedures result in the same value for sum of squares.

Deviation Method

Compute deviations of all scores from the mean. Square all deviations (x^2) and sum them (Σx^2) as follows

score	-	mean =	deviation	squared
10	-	30	-20	400
20	-	30	-10	100
30	-	30	0	0
40	-	30	10	100
50	-	30	20	400
			$\Sigma x=0$	Σx^2 = **1000**

Large groups will have a larger sum of squares than small groups, simply because there are more deviations in a large group. Dividing by N eliminates size of group from the result. This gives a truer picture of spreadedness in a group of numbers no matter how many are in the group. Divide the sum of squares by N in order to factor out the variable of group size. The resulting value is called the ***variance*** of the scores, and is symbolized by the lower case Greek letter sigma (**σ**).

$$\text{Variance } (\sigma^2) = \Sigma x^2 / N = 1000/5 = \underline{200.0}$$

Since we squared deviations before adding them, variance measures variability in *squared units.* It would be better if score variability were in the same unit of measure as the scores themselves. We can "undo" the squaring by taking the square root (**√**)[1] of the variance, like this:

$$\text{Standard Deviation } (\sigma) = \sqrt{\sigma^2} = \sqrt{200} = \underline{14.14}$$

The number 14.14 represents the standardized measure of variability for our example. This number represents, in the same unit of measure as our scores, the degree of spread-out-ness of the scores from the mean. The larger the number, the greater the spread. It is useful in comparing the variabilities in different groups of scores, but will become more meaningful in future statistical procedures.

This *deviation method* shows you exactly what a "standard deviation" is, and is fine to use when you have a few scores and a whole number mean. But if you have a large data set, and the mean is a fraction, like 73.031, computing individual deviation scores, squaring them, and then summing them can be painfully tedious. A *simpler way* to compute the sum of squares -- and get the very same result -- is to use the raw score formula.

Raw Score Method

The raw score method uses the *squares of each raw score* (rather than squares of deviation scores) to produce the sum of squares. The raw score formula for sum of squares is:

$$\Sigma x^2 = \Sigma X^2 - (\Sigma X)^2/N$$

where **ΣX^2** refers to the sum of squared-raw-scores (square all the scores and sum them) and **$(\Sigma X)^2$** refers to the sum-of-all-scores squared (sum the scores and then square the sum). Let's apply this formula to the same data that we used under the deviation method. We should get the same answer: $\Sigma x^2 = 1000$.

X	X^2
10	100
20	400
30	900
40	1600
50	2500
$\Sigma X = 150$	$\Sigma X^2 = 5500$
$(\Sigma X)^2 = 22500$	

$$\begin{aligned}\Sigma x^2 &= \Sigma X^2 - (\Sigma X)^2/N \\ &= 5500 - 22500/5 \\ &= 5500 - 4500 \\ \Sigma x^2 &= \underline{1000}\end{aligned}$$

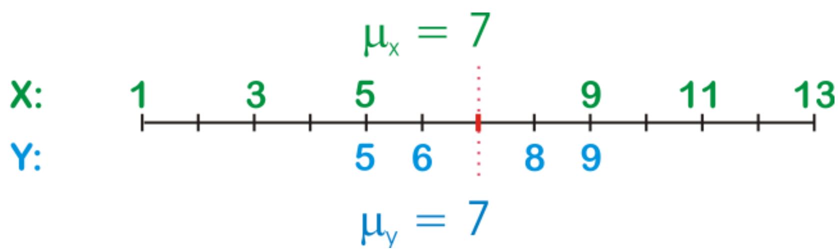
Warning:
There is a great difference
between **ΣX^2** and **Σx^2**.
Do not confuse the two!

[1]The "square root" symbol actually looks like this: $\sqrt{x}$ but is difficult to produce within the text. So I am using the simpler (√x) symbol. Later, with more complicated formulas, I will use graphical characters to indicate square root.

As you can see, both methods give sum of squares values of 1000. The raw score method is easier to do and less prone to arithmetic errors.

Equal Means, Unequal Standard Deviations

Let's say I have two groups of scores. The first group consists of scores **1, 3, 5, 9, 11 and 13**. We'll use the letter "X" to refer to them. The second set of scores are **5, 6, 8 and 9**. We'll use the letter "Y" to refer to them. I've put them on a scale below like this:



Notice that the **means of the two groups are equal**. But the degree of scatter (variability) among the scores is not. Let's compute the standard deviations of both groups to compare them. ***Which group <u>should</u> have the larger standard deviation?***[2]

Using the **<u>deviation method</u>**, we calculate the sum of squares of X as follows:

i	X_i	x_i	x_i^2
1	1	-6	36
2	3	-4	16
3	5	-2	4
4	9	2	4
5	11	4	16
6	13	6	36
N = 6	ΣX = 42	Σx = 0	Σx^2 = 112

The **variance** of group X equals $\Sigma x^2/N = 112/6 =$ **18.66**.
The **standard deviation** is the square root of variance, or $\sqrt{18.66}$ = **4.32.**

Using the **<u>raw score method</u>**, we calculate the sum of squares for Group X as follows:

i	X_i	X_i^2
1	1	1
2	3	9
3	5	25
4	9	81
5	11	121
6	13	169
n = 6	ΣX = 42	ΣX^2 = 406

$$\Sigma x^2 = \Sigma X^2 - (\Sigma X)^2/N = 406 - (42)^2/6 = 406 - 294 = \mathbf{112}$$

We get the same result, <u>112</u>, with either method.

[2] Did you say the X's? Good. You can see from the graph that the X's are spread out more than the Y's (another way of saying this is that the range of X is greater than the range of Y). We would expect the X's to have more variability than the Y's, and, in turn, the standard deviation of the X's will be greater.

Now let's compute variance and standard deviation for Group Y, which should produce a smaller sum of squares, variance and standard deviation than Group X did.

Here's the **deviation method**:

i	Y_i	y_i	y_i^2
1	5	-2	4
2	6	-1	1
3	8	1	1
4	9	2	4
$N = 4$	$\Sigma Y = 28$	$\Sigma y = 0$	$\Sigma y^2 = 10$

The variance of group Y equals $\Sigma y^2/N = 10/4 =$ **2.5**.
The standard deviation is the square root of variance, or $\sqrt{2.5}$ = **1.58.**

Using the **raw score method**, we calculate the sum of squares for Group 2 as follows:

i	Y_i	Y_i^2
1	1	1
2	6	36
3	8	64
4	9	81
$n = 4$	$\Sigma Y = 28$	$\Sigma Y^2 = 206$

$$\mathbf{\Sigma y^2} = \Sigma Y^2 - (\Sigma Y)^2/N = 206 - (28)^2/4 = 206 - 196 \mathbf{= 10}$$

Again, we get the same result, 10, with either method. Since the sum of squares equals 10, variance equals 2.5 and standard deviation 1.58, as calculated above. The deviation method illustrates the meaning of standard deviation, the raw score method gives the same result more simply.

We have computed the standard deviation for both groups of scores. The groups have identical means, but different "spreads." We expected that the scores of Group X would have the larger standard deviation than Group Y because of its larger spread of the scores. Calculations demonstrated a standard deviation of **4.32 in Group X** and of **1.58 in Group Y**, confirming our expectation.

Parameters and Statistics

So far we've used the symbol $\boldsymbol{\mu}$ to refer to the mean, $\boldsymbol{\sigma^2}$ to refer to the variance, and $\boldsymbol{\sigma}$ to refer to the standard deviation of a group of scores. We have treated these groups as populations. You will recall from Chapter 8 that a ***population*** of scores includes all the scores or subjects in a specified group (e.g., 7,000 Texas Baptist pastors). A ***sample*** is a subset of scores drawn from the population we wish to study (e.g., 700 randomly chosen Texas Baptist pastors).

We can compute mean and standard deviation for the population directly. The resulting values are called ***population parameters*** and are defined as $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$. We can compute mean and standard deviation for the sample directly. These values are called ***sample statistics*** and are defined as $\overline{X}$ and $\hat{\sigma}$. We can estimate population parameters from a sample of scores. These values are called ***estimated parameters*** and are defined as $\overline{X}$ and **s**. Let's illustrate these three sets of values.

Population Parameters

Suppose we have a population of 10,000 ministers. We want to compute the mean and standard deviation of their IQ. In order to compute these population parameters directly, you give all 10,000 ministers an IQ test. Sum the 10,000 IQ scores (ΣX), and divide by 10000 (N). The result is the *population mean*, symbolized by $\boldsymbol{\mu}$ (pronounced "myoo").

Subtract $\boldsymbol{\mu}$ from the 10,000 IQs ($x=X-\mu$), square the 10,000 deviations (x^2), sum them (Σx^2), divide by 10,000 (N), and finally take the square root ($\surd$). This yields the *population standard deviation* , symbolized by $\boldsymbol{\sigma}$.

Sample Statistics

The cost in time and materials to test 10,000 subjects and compute the parameters is not practical. Draw a random sample of 100 ministers (1%) and measure their IQs. Sum the 100 IQs (SX) and divide by 100 (N) to produce the sample mean. The *sample mean* is symbolized by $\overline{X}$, pronounced "X-bar".

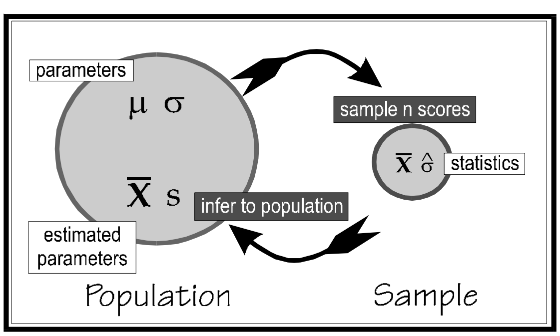
Subtract $\overline{X}$ from the 100 IQs ($x=X-\overline{X}$), square the 100 deviations (x^2), sum them (Σx^2), divide by 100 (N), and finally take the square root ($\surd$). This *sample standard deviation* is symbolized by a sigma with a hat on top ($\hat{\sigma}$), pronounced "sigma-hat."[3]

Estimated Parameters

When we can not compute population parameters directly, we must estimate them from sample statistics. This is not a problem for the estimate of the mean (μ). The sample mean ($\overline{X}$) is the best estimate. But due to the smaller number of scores in the sample -- because it is a subset of the population -- then the sample standard deviation ($\hat{\sigma}$) always underestimates σ.

This underestimation of $\hat{\sigma}$ requires a small correction factor in the equation for *estimated standard deviation* (**s**). While the equations for $\boldsymbol{\sigma}$ and $\hat{\sigma}$ have sum of squares divided by N or n[4],the equation for *estimated standard deviation* (s) has sum of squares divided by **n-1**.

Why n-1? It has to do with the Central Limit Theorem and you really don't want to know. (Okay, for those who do: the selection of a sample of n scores from the population reduces by one the number of n-sized samples that can be drawn from the



Population Parameters

$$\mu = \frac{\Sigma X}{N} \qquad \sigma = \sqrt{\frac{\Sigma x^2}{N}}$$

Sample Statistics

$$\overline{X} = \frac{\Sigma X}{n} \qquad \hat{\sigma} = \sqrt{\frac{\Sigma x^2}{n}}$$

Estimated Population Parameters

$$\overline{X} = \frac{\Sigma X}{n} \qquad s = \sqrt{\frac{\Sigma x^2}{n-1}}$$

[3]Some textbooks refer to the sample standard deviation as "sigma-tilde" ($\tilde{\sigma}$).

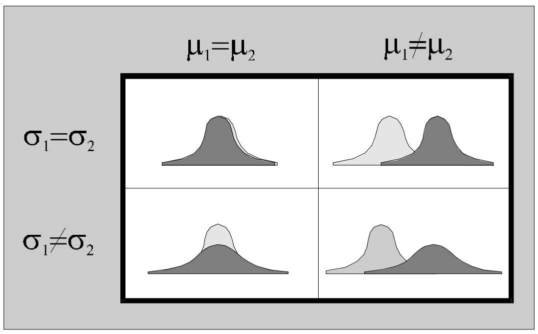
[4]Often, N and n are used interchangibly to refer to the number of scores in a set. Other times, N refers to the number of scores in a population, and n to the number of scores in a sample.

population. This reduces the number of degrees of freedom of the population by one. We'll talk more about degrees of freedom in a few chapters).

So, we have three sets of formulas. Mean and standard deviation are common concepts across the three versions, but there are important differences to note. Notice the use of "N" for parameters and "n" for samples. ***Match the formulas*** *with the* ***diagram*** *above.*

Standard (z-) Scores

We have demonstrated two related but separate characteristics of data sets. The first is central tendency (the ***mean is the most important measure***). The second is variability (the ***standard deviation is the most important measure***). Given two sets of data, there are four possibilities of comparisons. *Using the chart at left, write out the four possibilities in English.*[5]



Comparing two sets of scores is difficult because the groups usually possess different values of locus and scatter. Where do we begin? What is required is a **standard scale** which reflects in one value both mean and standard deviation. Then translating raw scores from each set to a single standard form would allow us to compare them directly. Direct comparison is possible because **transformed** scores from the groups have common "standardized" values.

The standardized score which reflects in one value both mean and the standard deviation of a set of scores is called a ***z-score***.

A raw score (**X**) from a population that has a mean of μ and standard deviation of σ is transformed into a standardized scale score (**z**) with this formula.

$$z = \frac{X-\mu}{\sigma}$$

The equation is pronounced ***z equals X minus mu over sigma***. In English, the formula means that a *standardized score is equal to a raw score minus the population mean divided by population standard deviation*.

A raw score (**X**) from a sample that has a mean of $\overline{X}$ and estimated standard deviation of **s** is transformed into a standardized scale score (**z**) with the following formula.

$$z = \frac{X-\overline{X}}{s}$$

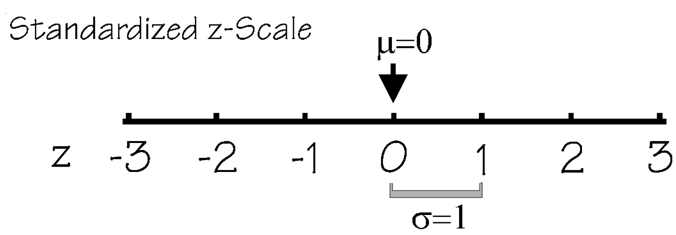
The equation is pronounced ***z equals X minus X-bar over s***. In English, the formula means that *a standardized score is equal to a raw score minus the sample mean divided by the estimated standard deviation.*

Both formulas reflect the same relationship between a raw score and a standard-

[5]Upper left: Two distributions have the same mean and standard deviation.
Upper right: Two distributions have the same standard deviation, but different means.
Lower left: Two distributions have the same mean, but different standard deviations.
Lower right: Two distributions have different means and standard deviations.

ized score in a distribution of numbers. The distinction is whether the distribution is a sample or a population.

Notice the values for mean and standard deviation are both part of the transformation formula. No matter what these parameters are, the standardized scores are plotted on a z-scale which looks like this:



For a standardized scale, the **<u>mean is always zero</u>** and **<u>standard deviation is always one</u>**. The z-score equations transform any group of scores into these standardized values. Let's look at an example of how z-scores facilitate comparison between scores.

John is taking Hebrew and Research.
On his midterm exams, he made a 85 in Hebrew and an 80 in Research.
On which exam did he do better?

It seems obvious that he did better in Hebrew than he did in Research. But the real answer is not so easy. To compare his performance on the two tests, we must take into consideration how well his classmates as a whole did. That is, we need to know the means and standard deviations for the two exams. Here's the information we need:

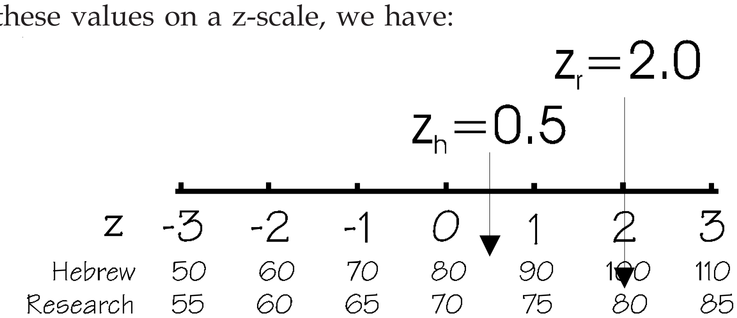
	μ	σ
Hebrew	80	10
Research	70	5

Now compute z-scores for Hebrew (z_h) and Research (z_r).

$$z_h = \frac{X_h - \mu_h}{\sigma_h} = \frac{85-80}{10} = 0.50$$

$$z_r = \frac{X_r - \mu_r}{\sigma_r} = \frac{80-70}{5} = 2.00$$

Placing these values on a z-scale, we have:



Notice several things about the diagram above. **<u>First</u>**, since the z-scores from Hebrew and Research now fall on the same standardized scale, we can ***directly***

compare them. It is clear from the scale that John did much better in Research, scoring two standard deviations above the mean, than he did in Hebrew, where he scored only one-half standard deviation above the mean.

Second, notice that the ***means of both classes line up on a z-score of 0***. In standardized scores, the mean is always 0.

Third, notice that the **σ** of 1 on the z-scale is equivalent to ***10 in Hebrew*** and ***5 in Research***.

Fourth, notice that John's score of ***85 in Hebrew falls directly below 0.50 on the z-scale***. His score of ***80 in Research falls directly below 2.00 on the z-scale***. Standardized scores lie at the heart of inferential statistics. These basic building blocks provide the foundation for procedures we'll study soon.

Summary

The three measures of **central tendency** are the ***mode, the median, and the mean***. These refer, respectively, to the most frequent score, the middlemost score, and the arithmetic average of a group of scores. In terms of statistical analysis, the mean is by far the most important of the three, and the most affected by skewed distributions.

Three measures of **variability** are the ***range, average deviation, and standard deviation***. The standard deviation (and its squared cousin, variance) is the most important of the three.

The two characteristics of mean and standard deviation can be combined to transform a raw score (**X**) into a standard score (***z***). Z-scores can be directly compared across groups, regardless of differing parameters.

Example

In my Ed.D. dissertation, I analyzed how much learning of the doctrine of the Trinity in Southern Baptist adults occurred over a seven week course. Cognitive tests were given at the beginning (Test 1), end (Test 2), end plus three months (Test 3) and end plus six months (Test 4).[6] I was also interested in whether the mental abilities of the three groups were balanced. Here is one of my Tables showing the means and standard deviations of these groups.[7]

You can notice several things immediately from the numbers below. The three groups' average mental ability, measured by the Otis-Lennon Mental Ability Test (maximum score: 80), were within 0.90 points of each other. All three groups learned a great deal about the doctrine of the Trinity -- all three groups jumped an average of 50.69 points over the seven weeks (Test #2 Total N minus Test #1 Total N). All three groups forgot some of what they learned, dropping an average of 11.48 points over three months and 17.98 points over six months.

Are these means *significantly* different? We will learn how to answer this question in Chapter 20.

[6]William R. Yount, "A Critical Comparison of Three Specified Approaches to Teaching Based on the Principles of B. F. Skinner's Operant Conditioning and Jerome Bruner's Discovery Approach in Teaching the Cognitive Content of a Selected Theological Concept to Volunteer Adult Learners in the Local Church," (Fort Worth: Southwestern Baptist Theological Seminary, 1978). 41-42

[7]Ibid., 168

APPENDIX XI

Means and Standard Deviation Scores

	Total N	X	Y	Z
MENTAL ABILITY	59.96* 15.58+	59.71 16.55	59.67 19.13	60.57 11.30
TEST #1	24.70 8.01	25.57 4.79	23.44 8.80	25.43 10.26
TEST #2	75.39 15.40	81.43 8.02	78.44 14.85	65.43 18.41
TEST #3	63.91 13.91	66.00 12.36	67.78 15.97	56.86 11.44
TEST #4	57.41 11.56	61.00 9.81	59.22 11.58	52.29 11.86

*Mean +Standard Deviation

Vocabulary

$\overline{X}$	X-bar, the average or mean of a group of scores (sample)
μ	the average or mean of a group of scores (population)
σ^2	sigma-squared, the population variance
σ	sigma, the population standard deviation
$\hat{\sigma}^2$	sigma-hat squared, sample variance
$\hat{\sigma}$	sigma-hat, sample standard deviation
average deviation	|x|/n : Sum absolute values of deviation scores, then divide by n
average	sum of scores divided by the number of scores
central tendency	focal point of scores: mean, median, mode
estimated parameter	$\overline{X}$ and s, computed from sample, infers population parameters
mean	average score
median	middlemost score
mode	most frequent score
n	number of scores (sometimes used to refer to one group within experiment)
N	number of scores (sometimes used to mean entire experiment)
parameter	population measurements (μ, σ)
range	distance between highest and lowest scores in a group
standard deviation	standardized measure of variation in scores: s
statistics	sample measurements ($\overline{X}$ and $\hat{\sigma}$)
sum of squares	sum of squared deviation scores
variability	measure of spreadedness in a group of scores
variance	measure of spreadedness in squared units
x	deviation score: difference between score (X) and mean (μ or $\overline{X}$)
X	raw score: e.g., test score
z-score	standardardized score which reflects both μ and σ (or $\overline{X}$ and s)

[8]Ibid., 169

Study Questions

1. What are the modes for the sets of scores below?

a. 1 2 3 4 5 6 6 7 8 9 — Mode: _____

b. 1 2 3 4 5 6 6 7 8 8 — Mode: _____

c. 1 1 2 2 3 3 4 4 5 5 — Mode: _____

2. What are the medians for the following data sets?

a.	10	15	20	22	27	29	33		Md: _____
b.	3	7	78	45	2	56	4	7	Md: _____

3. Compute the mean, sum of squares (use deviation method), variance and standard deviation for the following scores:

65 70 70 75 85 90 95

4. Using the scores in #3, compute the sum of squares with the raw score method.

5. You have taken midterm exams. Your score in New Testament survey was 75. Your score in Principles of Teaching was 90.

	n	ΣX	Σx^2
NTS	100	7020	2500
PT	25	2175	225

a. Compute means for both classes.

b. Compute standard deviations (s) for both classes.

c. Transform your midterm scores into z-scores.

d. Plot your standard scores on a z-scale. Include the appropriate raw score scale values for the two classes.

e. In which class did you do better? Explain how you know this.

Sample Test Questions

1. The most important measure of central tendency is the
   A. mode
   B. mean
   C. kurtosis
   D. range

2. The measure of central tendency which behaves like a balance or a teeter-totter is the
   A. mean
   B. mode
   C. median
   D. range

3. If you add together all the deviations of scores about the mean, the result is
   A. the standard deviation
   B. the variance
   C. the sum of squares
   D. zero

4. You compute mean and median of a distribution and find that the median is larger. You know from this that the distribution is
   A. normal
   B. leptokurtic
   C. positively skewed
   D. negatively skewed

4. Parameters are to statistics as
   A. mean is to variance
   B. population is to sample
   C. average deviation is to standard deviation
   D. Greek is to English

5. The mean and standard deviation of the z-scale is, respectively,
   A. 1, 1
   B. 0, 1
   C. 1, 0
   D. 0, 0

