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Basic Math Skills

Mathematical Symbols
Mathematical Operations

Learning to communicate in statistics is very much like learning a foreign language. First, there is a new alphabet. Then you learn that these new letters form words which mean specific things. Then you learn how to put words together to express ideas clearly and correctly.

Statistics has its parallel in a new alphabet of mathematical symbols, and specific terms to refer to statistical realities, and finally phrases which refer to statistical procedures and outcomes. We introduce many of these terms, chapter by chapter, in the statistics portion of the text. In this chapter we will introduce you to basic mathematical symbols and terms that you'll use as building blocks for the statistical procedures.

In this review we focus on mathematical symbols, fractions, negative numbers, percents and proportions, exponents, square roots, and simple algebraic solutions.

Mathematical Symbols

Mathematical symbols are characters which convey specific operations in shorthand form. Mathematical symbols include arithmetic operators, the square symbol, the square root symbol, the sum symbol, and parentheses and brackets.

Arithmetic Operators

The simple arithmetic symbols used in this text are addition (+), substraction (-), multiplication (·), and division (/, or a horizontal line).

$$6 + 3 = 9 \qquad 6 - 3 = 3$$
$$6 \cdot 3 = 18 \qquad 6 / 3 = 2$$

Sometimes, you'll see the lower case "x" used to indicate multiplication, as in "6 x 3 = 18."

Square (^{2})

The "square" of a number is that number multiplied by itself. We represent a value squared by way of the superscript "2." So,

$$3^2 = 3 \cdot 3 = 9$$

The number 9 is the square of 3. Or, "9 is 3 squared." We can also use the superscript "3" to indicate "cubing" a number -- multiplying itself three times. Or the superscript "n" to indicate multiplying a number by itself "n" times.

Square Root (√)

The square root symbol is used to represent the reverse of squaring. When we enclose a number or formula within the square root symbol, we are asking for the number which, when multiplied by itself, gives us the value under the square root. Using the previous example, we can say that 3 is the square root of 9. Or using the symbol above, $\sqrt{9} = 3$.

Another form of this symbol is this: $\sqrt[2]{9} = 3$. We can extend the symbol to include the "cube root" of a number: $\sqrt[3]{27} = 3$. Or the "n"th root of a number: $\sqrt[n]{X} = Y$.

The Sum Symbol (Σ)

The capitol sigma means to add up. In its simplest form it looks like this: ΣX. The English letter X refers to scores on a test. So, ΣX means to "add together all the test scores," and is read "sum of X."

In its full form, subscript notation is used to be more mathematically precise. Our simple "ΣX" looks like this in full dress:

$$\sum_{i=1}^{n} X_i$$

and reads "the sum of X sub i, from i equals 1, to n."

Given a series of numbers 13, 56, and 5, X_1 equals 13, X_2 equals 56, and X_3 equals 5. The letter "i" refers to the subscripts 1, 2, and 3. The result of the Σ operator is 74, the sum of these three numbers. In most of our discussion, we'll use the simpler ΣX.

Parentheses and Brackets

Parentheses ***( )*** and brackets ***[ ]*** are sometimes used in large equations to organize values properly. If I asked you what this simple equation equalled, what would you say?

$$2 / 4 + 1 = ?$$

Did you say ***1.5***? You're right! Did you say ***0.40***? You're right!

Well, it depends on whether you do the division first (.5 + 1) or the addition first (2/5). Actually, "convention" says to multiply and divide before adding or subtracting. But *parentheses* are used to group numbers so that they are calculated correctly, without confusion. Now, what is the answer to this one?

$$(3 / 9) + 2 = ?$$

The parentheses tell you to divide (3/9) first, then add 2. Your answer should be 2 1/3, or in decimal form, 2.333. What about this one?

$$[(1/2)+(3/4)] \cdot [2/3]$$

Compute the inside parentheses ( ) first, then the brackets [ ].

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(1/2) = 0.50
      (3/4) = 0.75
            0.50 + 0.75 = 1.25
                  2/3 = 0.666
                        1.25 · 0.666 =  0.8325
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Work through equations from the innermost parentheses or brackets outward to insure that arithmetic operations are done properly.

$$\frac{a(b - c)}{a^2 - c^2}$$

Using Letters as Numbers

Mathematical formulas or equations use letters to establish a general relationship among the variables in the equation. You then "solve" the equation by replacing the letters with their appropriate numbers. Look at the example at right:

$$\frac{2(7 - 1)}{2^2 - 1^2}$$

This formula sets the specific relationship among variables a, b, and c. When the actual numbers are inserted into the equation for a, b, and c, the equation can be solved. Let's say that a = 2, b = 7, and c = 1.

Now let's solve this equation step by step.

1. Compute the value inside the parentheses first.
   The letter "b" is 7, and "c" is 1, so (b - c) = (7 - 1) = 6.

2. Multiply (b - c) by a.
   Since a = 2 and (b - c) = 6, the result of this multiplication is 12.

3. Since a = 2, aI = 4. Since c = 1, cI = 1.
   Therefore, the term aI - cI = 4 - 1 = 3.

4. The final step is to divide the numerator, or top part of the equation (12) by the denominator (3), which equals 4. given a = 2, b = 7, and c = 1.

Simply consider letters as empty place-holders in a formula. Formulas can be solved only as the place-holders are given real values. But formulas demonstrate simply and efficiently the relationships among the place-holder variables.

Mathematical Concepts

The mathematical concepts in this section include fractions, negative numbers, percents and proportions, exponents, and some simple algebra.

Fractions

A fraction is a number that represents a part of a whole. One-half. Three-fourths. Seven-eighths. Fractions are a common element in statistical formulas. Simplify your calculations by converting fractions to decimals. Do this by dividing the numerator (top number) by the denominator (bottom number). Then apply the mathematical operations (+, -, · or x, /, ^{2}, √). For example:

1/4 - 1/6	= 0.250 - 0.166	= 0.084
2/3 + 4/5	= 0.666 + 0.800	= 1.466
7/9 · 11/16	= 0.777 · 0.687	= 0.534
√(2/3)	= √.666	= 0.816

Negative numbers

Negative numbers are indicated by the presence of a minus (-) sign. Specific rules govern the handling of negative numbers in formulas.

Multiplying a negative by a positive number yields a negative: $-1 \cdot 3 = -3$

Multiplying two negatives yields a positive result: $-1 \cdot -3 = 3$

Dividing a negative number by a positive number, or dividing a positive number by a negative number yields a negative number. $-6 / 2 = -3$

Dividing two negative numbers yields a positive. $-6 / -2 = 3$

Squaring a negative number yields a positive result. $(-3)^2 = 9$

Taking the square root of a negative number yields an *imaginary* number (since multiplying a negative by a negative produces a positive) -- which we will not be using in this text.

Percents and Proportions

A decimal fraction is a part of a whole in a decimal form. The whole number fraction of 2/3 is equal to the decimal fraction of 0.666. A ***proportion*** is merely another name for a decimal fraction. "Five is what proportion of ten?" **(0.50)**

A ***percentage*** is equal to 100 times proportion followed by a percent sign (%). The proportion of 0.5 is equivalent to **50%**.

Both terms refer to the same concept. *If three-fourths of your questionnaires* are returned, then you have a proportion of 0.75 and a percentage of 75%.

Exponents

We have already defined the "square" of a number. Squaring a number is a simple example of raising that number to the power of 2, where "2" is the exponent. But as we have already seen, exponents can be any number, such as these examples show:

$$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$$

We've already discussed the fact that letters can replace numbers in formulas. This is also true of exponents. Look at this:

$$p^n = ? \quad (p=4 \text{ and } n=3)$$

The answer is 64. Substituting the numbers for the letters gives us

$$4^3 = 4 \cdot 4 \cdot 4 = 64$$

Simple Algebra

On the previous page we solved a formula by substituting numbers for all three letters. When a = 2, b = 7, and c = 1, the formula equals 4, which we'll call "d." But we can also manipulate the letters in the equation to "solve for another variable." Let's say we have a formula containing a, b, c, which equals d. ***How do we change the formula so that it provides the proper relationship among a, c, and d to equal b?*** The basic rule in algebraic manipulation is "whatever is done to one side of an equation (one side of the "=" sign) must be done to the other." Let's move **step by step** through the transformation.

$$\frac{a(b-1)}{a^2 - c^2} = d$$

Step 1. Isolate the numerator where b is. If we multiply the right side of the equation by (aI-cI) then this would cancel the (aI-cI) term in the denominator. This is because any number or term divided by itself equals 1. But to multiply the right side by (aI-cI), we must multiply the left side by the same term. Therefore, we get

$$\frac{(a^2 - c^2) \cdot a(b-1)}{(a^2 - c^2)} = d(a^2 - c^2)$$

Step 2. Canceling out the two terms $(a^2 - c^2)$ on the left above we have

$$a(b-1) = d(a^2 - c^2)$$

Step 3. Now to isolate the (b-1) term from the left side of the equation, divide both sides by a.

$$\frac{a(b-1)}{a} = \frac{d(a^2 - c^2)}{a}$$

Step 4. Canceling the a's in numerator and denominator (a/a =1) we have

$$b - 1 = \frac{d(a^2 - c^2)}{a}$$

Step 5. Now to isolate b from the b-1 term, we must add 1 to both sides of the equation.

$$b - 1 + 1 = \frac{d(a^2 - c^2)}{a} + 1$$

Step 6. This final operation yields the following formula:

$$b = 1 + \frac{d(a^2 - c^2)}{a}$$

Our equation has been transformed from one solved for "d" to one solved for "b." It is the same formula and denotes the same relationships among the variables. We can test this by solving for b, given a, c, and d. (The answer is 7, given a = 2, c = 1, and d = 4.

$$b = 1 + \frac{4(2^2 - 1^2)}{2}$$

b = 7

...which it, in fact, does. So our transformation is correct.

Summary

The basic terminology covered in this chapter are essential building blocks to the procedures we'll discuss later in the course. It will help you to fully understand this chapter before proceeding into statistical formulas. If you must stop several times per page to look up some term or symbol that you've forgotten, you will become very frustrated. Commit the terms in this chapter to memory. Review them often. Let them become a regular part of your vocabulary. If you do, you'll find your study of statistics much more enjoyable and rewarding.

Vocabulary

exponent	"n" in the expression X^n: multiply x by itself n times. $9^3 = 9•9•9 = 729$	
percent	(x/y)•100 = % :	e.g., *7 is what* ***percent*** *of 11?* : (7/11)•100 = 63.63%
proportion	(x/y) = p :	e.g., *7 is what* ***proportion*** *of 11?* : (7/11) = 0.636
square root	$\sqrt{x}$	: What number multiplied by itself = x? : $\sqrt{9} = 3$
square symbol	X^2	: Multiply x by itself : $13^2 = 13•13 = 169$
sum symbol	ΣX	: Add scores in group : e.g., (2, 4, 5) $\Sigma X = 2 + 4 + 5 = 11$

Study Questions

Calculate the following arithmetic expressions:

1. (-3 x (6 - 5)) - 9 =
2. (3)/(-4 x -2) =
3. q^n where q=5 and n=3
4. 3/8 - 1/3 + (-2/5) =
5. 9 is what percent of 7?
6. 6 is what percent of 9?
7. 13 is what proportion of 15?

Solve the following algebraic expressions:

8. Solve for X: Z(X-W) = (1-C)
9. Solve for A: B(C-1) = (AB-1)/3